Grade 8

CONTENT BOOKLETS: TARGETED SUPPORT

Term 1

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). The FSS helped the DBE trial the NECT Maths, Science and language learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Provincialisation Programme. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

CONTENTS

TOPIC 1: WHOLE NUMBERS

INTRODUCTION

- This unit runs for 6 hours, approximately 10 lessons.
- It is part of the Content Area ‘Numbers, Operations and Relations’ which counts for 30% in the final exam.
- The unit mainly covers work that the learners have been doing throughout primary school. Some concepts in the Finance section are new to them so more time can be allocated to this.
- The purpose of consolidating this section again is to ensure that the learners are confident with calculations and order of operations (BODMAS). These rules are used in every aspect of mathematics until Grade 12.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Mental Calculations • Ordering and Comparing Whole Numbers • Properties of Whole Numbers • Calculations using Whole Numbers • Calculation techniques • Multiples and Factors • Problem Solving	• Mental Calculations • Ordering and Comparing Whole Numbers • Properties of Whole Numbers (recognise division by zero property) • Calculations using Whole Numbers • Calculation techniques • Multiples and Factors (prime factors) • Problem Solving (increasing or decreasing in a given ratio) • Hire purchase and exchange rates included in financial Mathematics	• Properties of Whole Numbers • Calculations using Whole Numbers • Calculation techniques • Multiples and Factors • Problem Solving • Commission, rentals and compound interest included in financial mathematics

GLOSSARY OF TERMS

Term	Explanation / Diagram
Whole Numbers	The numbers 0 ; 1 ; 2; ; 3; ; 4......
Natural Numbers	The numbers 1 ; 2 ; 3 ; 4 ....
BODMAS	The order of operations that always needs to be followed. Brackets ; Of ; Multiply and Divide ; Add and Subtract
Inverse operation	The opposite operation that would 'undo' the last operation. Addition and Subtraction are the inverse of each other. Multiplication and Division are the inverse of each other.
Factors	A number that can go into another number without a remainder.
Multiples	A multiple of a number is that number multiplied by a natural number. Commonly known as the 'times tables'
Prime Numbers	A number that has only two factors – one and itself. The first five prime numbers are: 2 ; 3 ; 5 ; 7 ; 11
Composite Numbers	A number that has more than two factors. The first five composite numbers are: 4 ; 6 ; 8 ; 9 ; 10
Highest Common factor (HCF)	The highest factor that can go into two or more numbers that is common to both of them. For examples: the HCF of 10 and 15 is 5.
Lowest Common Multiple (LCM)	The lowest multiple of two or more numbers that is common to both of them. For example: the LCM of 8 and 6 is 24.
Commutative	An operation that works even if the numbers are swopped around. Example : 5 + 2 is the same as 2 + 5 Addition and multiplication are commutative. Division and subtraction are NOT commutative.
Associative	An operation that works if you start anywhere in the question. Example: 3 + 6 + 5 It makes no difference if you work left to right or first add 6 then 3 then 5 – you will get the same answer. Addition and multiplication are associative. Division and subtraction are NOT associative.
Distributive	Used to multiply when a bracket is involved. The number outside can be multiplied by EACH number inside instead of first working inside the bracket. The answer will be the same.
Less than (<)	Smaller than
Greater than (>)	Larger than or bigger than
Ascending	Going up. Smallest to biggest.
Descending	Going down. Biggest to smallest.
Ratio	A relationship between quantities of the same unit
Rate	A relationship between quantities of different units
Profit	Money made when income exceeds (is bigger than) expenditure. (You made more money than you spent).

Term	Explanation / Diagram
Loss	Money lost when expenditure exceeds income. (You spent more than you made).
Discount	Paying less than the usual price. A percentage of the original price is taken off.
Budget	A plan to manage money.
Loan	Borrowing of money.
Interest	The extra money paid back after taking a loan.
Simple Interest	The interest on a loan is calculated on a yearly basis.
Hire Purchase	A plan used to buy items from a shop and pay for it monthly.
Deposit	A sum of money paid as a first instalment on an item with the understanding that the balance will be paid at a later stage.
Exchange Rates	The rate of one country's money against another country's money.

SUMMARY OF KEY CONCEPTS

Ordering and Comparing Whole Numbers

Learners need to order and compare Whole Numbers. To do this they need to know the difference between < (less than) and > (greater than). An understanding of the words ascending and descending are also important.

Properties of Whole Numbers

1. Commutative Property
   This works for addition and multiplication.
   $5 + 2 = 2 + 5$ and $5 \times 2 = 2 \times 5$
   It does NOT work for subtraction and division
   $10 - 5 \neq 5 - 10$ and $10 \div 5 \neq 5 \div 10$

2. Associative Property
   This works for addition and multiplication.
   It means that if you are adding or multiplying more than two numbers, it doesn't matter where you start or which numbers you pair up first.

   Example: $4 + 2 + 6$ can be done by first adding the 4 and 6 and then the 2
   OR by first adding the 2 and 6 and then the 4 and so on.

   Either way your answer will still be 12.

   $4 \times 2 \times 6$ can be done in the same way and you will still get an answer of 48

3. Distributive Property
   This is used in multiplication only.

   Example: 2 (4 + 1)
   To do this, we would usually do the addition in brackets which (would give 5) then multiply by 2 (which would give 10) and this would be perfectly correct.

   Another way that could be used (and will be VERY useful later in Algebra), is to distribute the 2 into the bracket, by multiplying with each of the numbers inside the bracket.

   2 (4 + 1)
   = 8 + 2 (2 x 4 and 2 x 1)
   = 10

4. Zero

   Zero is a powerful number.

   In addition and subtraction:

   Adding or subtracting zero from any number will always give the same number back.

   Example: 10 + 0 = 10 15 - 0 = 15

   In multiplication:

   Multiplying a number by zero ALWAYS gives zero.

   Example: 5 x 0 = 0 (if you have no (zero) fives you must have nothing (zero))

   In division:

   Division by zero cannot be done. There is no solution. We say it is undefined.

   Example: 10 ÷ 0 = undefined

   Explanation: Remember that multiplication and division are inverse operations. A division sum should work backwards into a multiplication sum.

   Example: If 10 ÷ 2 = 5 then 5 x 2 = 10

   So, whatever a number divided by zero is equal to, should multiply by zero give that number back. And there is no number that you can multiply by zero to give another number.

   20 ÷ 0 = ------ (no matter what you think goes here, it can never multiply by zero to give the 20 back)

   ∴ 20 ÷ 0 = undifined

Multiples and Factors

1. Factor

   A number that can go into another number without a remainder.

   2 is a factor of 10

   3 is NOT a factor of 10

   The factors of 10 are: 1 ; 2 ; 5 ; 10

2. Multiple

   Multiples can be found by multiplying a whole number with all the natural numbers. These are more commonly known as times tables.

   10 is a multiple of 2

   Multiples of 2 are: 2 ; 4 ; 6 ; 8……

   (2x1) (2x2) (2x3) (2x4)

3. Prime Number
   A prime number is a number that has only two factors – one and itself.
   5 is a prime number because its only factors are 1 and 5.

4. Composite Number
   A Composite Number has more than two factors.
   12 is a composite number as its factors are: 1 ; 2; 3; 4; 6; 12

5. Writing a Composite Number as a product of its prime factors
   Learners need to do this without the use of a calculator. The ladder method is used in this case. Only prime numbers can be used to divide so a knowledge of prime numbers is essential. It is best that learners know the first five prime numbers well (2 ;3 ;5 ;7 ;11) so they can write them down before they start one of these questions.

Example: Write 200 as a product of its prime factors.

2	200
2	100
2	50
5	25
5	5
	1

$\therefore$ 200 = 2 x 2 x 2 x 5 x 5 = $2^3 \times 5^2$

6. Highest Common Factor (HCF) and Lowest Common Multiple (LCM)
   The quickest way to do these is to write the numbers involved as a product of their prime factors.
   The same numbers will be used to explain both questions.

Example: Find the HCF and LCM of 8 and 12

First write each number as a product of its prime factors, using the ladder method if necessary.

8 = 2 x 2 x 2

12 = 2 x 2 x 3

HCF:

Circle all the factors that each number has in common (in this case, 2 x 2)

- Write these down and calculate the answer
  HCF : 2 x 2 = 4 (4 is the highest number that can go **into** 8 AND 12)

LCM

- Write one of the numbers' product of prime factors down (choose any one)
  2 x 2 x 2
- Now look at the other number's prime factors and include any factors not already listed
  2 x 2 x 2 x 3
  (we already had a 2 x 2 so needed to now include to the x3 to ensure we had all prime factors from the 12 listed)
- Calculate the answer
  LCM: 2 x 2 x 2 x 3 = 24
  (24 is the lowest number that 8 AND 12 can **both** go into)

Ratio and Rate

1. Ratio
   This is a comparison of two or more quantities that have the same unit. In other words, quantities of the same kind are being compared.

   Example:
   When mixing a dilute cool drink, the instruction may say, 'mix in a ratio of 1 : 3'. This means you will use 1 part juice to 3 parts water in the glass. It is important to note that there will be 4 parts liquid used in total.

   a. Sharing in a given ratio
   An amount can be shared among two or more people in a certain ratio.

   Example: R1 500 needs to be shared between 3 people in a ratio of 2 : 3 : 5
   This means that one person will get 2 parts of the money, another will get 3 parts and a 3rd person will get 5 parts.

   Step 1:
   Total the number of parts required.
   2 + 3 + 5 = 10

   Step 2:
   Change each part of the ratio given into a fraction, using the total found as the denominator.

   $\frac{2}{10}$ $\frac{3}{10}$ $\frac{5}{10}$

   Step 3:
   Calculate what portion of the money each person must get by multiplying each fraction by the total.

   $\frac{2}{10} \times 1\,500 = 300$ $\frac{3}{10} \times 1\,500 = 450$ $\frac{5}{10} \times 1\,500 = 750$

b. Increasing and decreasing in a given ratio
To increase or decrease a number in a given ratio, the ratio needs to be turned into a fraction and multiplied by the amount.

To increase: turn the ratio into an improper fraction (numerator larger than denominator)

To decrease: turn the ration into a common fraction (denominator larger than numerator)

Example: Increase 120 in a ratio of 2 : 5

$\frac{5}{2} \times 120 = 300$

Decrease 120 in a ratio of 2 : 5

$\frac{2}{5} \times 120 = 48$

2. Rate
A comparison of two quantities that have different units

Example: distance and time (speed) 35km/hr (35 kilometres per hour)

rands and kilograms (cost) R1,50/kg (R1,50 per kilogram)

Example: A driver covers a distance of 600km in 5 hours. What is his average speed?

Divide the distance by the time $600 \div 5 = 120$

His average speed was 120km/hr

Finance

Ensure learners have a clear understanding of the following terms. If possible, bring manipulatives in while teaching finance or ask learners to bring in a cell phone account for example. Get learners to draw up a budget if they get pocket money. Ask learners to look for advertisements where discounts are offered. Have discussions about how a small business would be run and what would need to be done to make a profit rather than a loss.

1. Budgets
   A budget is a plan of how money will be managed and spent. A budget is important – if a good budget is made and followed then a person is not likely to go into debt.
   "A budget is telling your money where to go instead of wondering where it went" (Dave Ramsey)

2. Accounts
   Some basic services like electricity, water and telephone are paid for in arrears (after the service has been used) and an account is sent to the customer, so he/she knows what to pay. Other services (like rent) are paid in advance (before the service has been used).
   It is also possible to have an account with a shop which allows a person to buy on credit and pay later. In this case interest (extra money) is charged. In other words, the item bought will cost more than it would have if cash had been paid.

3. Profit and Loss
   If a business makes more money than has been spent – there will be a profit
   If a business makes less money than has been spent – there will be a loss

4. VAT (Value added tax)
   This is a tax paid on almost every item you buy. The price tag always includes the tax, so you may not realise you are paying it. VAT in south Africa is 15%.

5. Loans
   When a person does not have the full amount to buy an item, it is possible to take a loan for the money needed. The loan would then be paid back (usually on a monthly basis) and interest would be added. All banks and some shops and private companies all offer loans.

6. Discount
   An amount (usually a percentage) offered off an original price.

Example: In a sale, all items in a shop are being sold with a discount of 15%. If an item was marked as costing R200, what price will you pay with the discount?

Step 1: Calculate the discount

15% of R200

$= \frac{15}{100} \times 200$

$= 30$

Step 2: Subtract this from the original price

R200 – R30 = R170

7. Percentage increase
   To find by what percentage an amount has been increased we need to find the difference in the two amounts (new and original) then divide it by the original amount. This figure multiplied by 100 will give the percentage.

$$\text{Percentage increade} = \frac{\text{actual increase}}{\text{original amount}} \times 100\%$$

The actual increase is: new amount subtract original amount.

This may be easier to remember as: $\frac{\text{new - old}}{\text{old}} \times 100 = \%$ increase

Example: A salary is increased from R10 000 to R11 500. Find the percentage increase.

$\frac{11500-10000}{10000} \times 100 = 15\%$ increase

8. Exchange rates
   If you visit another country, you would need to change your money (South African Rand) for that country's money. To do this, you need to know what the current (it changes) exchange rate is. (Remember: rate is a comparison of two different units).

   These are determined by the economy. Exchange rates can be very volatile and change continuously more than once per day. When dealing with an exchange rate it is very important that you know when you multiply by the rate and when you divide by the rate.

   Exchange rates are not only important for people lucky enough to go on holiday. They are important for businesses that either export their goods to other countries or import goods from other countries into South Africa. Exchange rates also affect the price of items.
   It helps to deal with exchange rates the same way you deal with ratios.

For Example: If the exchange rate is \$1 = R 9,75 then how many dollars will I get for R 1000?

Step 1: Write down the rate given: \$1 = R 9,75

Step 2: Write underneath the rate the other information given (ensuring the same currencies are written underneath each other)

? = R1000

Step 3: Consider the side with two pieces of information (in this case the rand value) and ask 'what operation was performed to get from one to the other (in this case, what do I do to R9,75 to get to 1000) – you can ONLY use multiplication or division with it being a ratio.

\$1 = R 9,75 (× 102,564…)

? = R1000 (calculated by dividing 1000 by 9,75)

Step 4: Multiply the other side by the same

(× 102,564) \$1 = R 9,75

\$ 102,56 = R1000

And how many Rands would I get if I had \$500?

Step 1: \$1 = R 9,75

Step 2: \$500 = ?

Step 3: \$1 = R 9,75

(x 500) \$500 = ?

Step 4: \$500 = R4875

9. Simple interest
   All loans always have interest added on. This is basically the fee paid for borrowing the money. It is usually a percentage and is mostly calculated on a yearly basis (in this case, it is simple interest). The good news is – when you save money you can earn interest.
   Simple interest works as follows: If you are borrowing money, a percentage of the amount borrowed is added to that amount annually. The same amount is added every year – even if a few years have passed and most of the loan may already have been paid back.

For example: if you borrow R 1000 and the interest charged is 12% p.a. (per annum or every year), how much would you pay back if:

a. you took 1 year to pay it back?

b. you took 5 years to pay it back?
   Solution: Interest paid: $P \times i \times n$ where P is the amount started with, i is the interest rate and n is the number of years. Once the interest has been found it will be added to the amount started with (P)

a. Interest: 12% of R1000 = $\frac{12}{100}$ × 1000 = 120

Using the formula: Interest = $P \times i \times n$ = 1000 × 0,12 × 1 = 120

Total amount to pay back: R1000 + R120 = R1120

Note that only 1 × 120 was added because the loan is only for 1 year.

b. Interest: 12% of R1000 = $\frac{12}{100}$ × 1000 = 120

Using the formula: Interest = $P \times i \times n$ = 1000 × 0,12 × 5 = 600

Total amount to pay back: R1000 + (R120 × 5) = R1000 + R600 = R1600

Note that this person will pay R600 interest as the R120 calculated is what

will be charged per year.

There is a formula that can be used so that you can do this whole calculation at once instead of finding the percentage, then multiplying it by the number of years then adding it to the original amount (so much work!)

Accumulated amount

Principal amount (the amount you started with)

$$A = P(1 + i.n)$$

Interest rate
(as a percentage/decimal)

Number of years
(this is only because you are paying it off annually
-n will not always represent years)

Using the distributive law, you should see why this formula works:

$$A = P(1 + i.n)$$

$$A = (P \times 1) + (P \times i \times n)$$

$P \times 1$: The amount we started with $P \times i \times n$: the interest

Added together – they make the final amount

10. Hire Purchase

If a shop gives a loan it is called hire purchase. If for example, if a person needed to buy a fridge but didn't have enough money, the shop would offer a hire purchase agreement where a deposit would be paid so the fridge could be taken immediately and the rest of the price of the fridge as well as some interest would be paid in monthly instalments over a given time period.

- Simple Interest is always used in Hire Purchase.
- A person can buy something from a shop on credit by paying a deposit and paying the balance (which will include interest) over several months.
- Why is it called hire purchase? Basically, the item bought is only being hired until it is fully paid for. If, for some reason, a payment was not made, the shop would be able to take the item back – then the amount that had been paid so far would just be a hiring fee.

An example of a hire purchase question using simple interest:

A computer costs R5999. The shop requires a 10% deposit and the rest will be paid in equal monthly instalments over a three-year period at an interest rate of 14%p.a. (per annum which means per year). Find the monthly instalments.

First calculate the deposit:
10% of 5999 = 599,90

Subtract this from the amount owing (because you have already paid it and don't owe as much anymore)

New amount owing is R5999 – R599,90 = 5399,10

Secondly, calculate the total amount now owing to the shop after the deposit has been taken off.

$A = P\ (\ 1 + i.n\ )$

A = 5399,10[1+ (0,14) (3)]

= 7666,72

Divide this amount owing by the number of months in the years to find the monthly repayment.

R7666,72 ÷ 36 = R212,96

TOPIC 2: INTEGERS

INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area 'Numbers, Operations and Relations' which counts for 25% in the final exam.
- The unit covers similar work to that which was covered in Grade 7.
- The purpose of practicing and understanding how to work with Integers is important for all aspects of mathematics. From Grade 8 onwards many steps in questions require the skill of working with negative numbers. It is also important in many real life situations especially when dealing with money and debt.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Counting, ordering and comparing integers • Calculations with Integers • Properties of Integers • Problem Solving	• Counting, ordering and comparing integers • Calculations with Integers (multiplication, division and a combination of all four operations, including squaring and cubing and square roots and cube roots) • Properties of Integers (recognize and use additive and multiplicative inverses) • Problem Solving (involving multiple operations)	• Calculations with Integers • Properties of Integers • Problem Solving

GLOSSARY OF TERMS

Term	Explanation / Diagram
Integer	The set of Integers is made up of all the whole numbers as well as the negative numbers. There are NO fractions in Integers. Integers consist of all the positive and negative counting numbers.
Negative	Numbers less than zero are shown by putting a minus sign in front. Example: -4 (minus 4 or negative 4) is 4 places below (smaller than) zero
Positive	Numbers greater than zero are all positive. The + sign is not necessary. Example: +5 = 5
Additive Inverse	Additive inverses are numbers exactly the same distance away from zero on either side of the number line. Example: 7 is the additive inverse of -7
Square	To multiply a number by itself Example: $3^2 = 3 \times 3 = 9$
Cube	the product of a number multiplied by its square, represented by a superscript figure 3. Example: $4^3 = 4 \times 4 \times 4 = 64$
Square Root	The opposite operation to squaring Example: $\sqrt[3]{25} = 5$ since 5 x 5 = 25
Cube Root	The opposite operation to cubing Example: $\sqrt{8} = 2$ since $2 \times 2 \times 2 = 2^3 = 8$
Ascending	Going up. From smallest to biggest.
Descending	Going down. From biggest to smallest.

SUMMARY OF KEY CONCEPTS

The set of Integers is an extension of the set of Whole Numbers.

Every positive whole number (every natural number) has an opposite number on the other side of zero.

These are the **additive inverses** and together, they form the set of Integers.

Example: -3 is the additive inverse of 3

9 is the additive inverse of -9

$Z \in$ {….-3 ; -2 ; -1 ;0 ;1 ;2 ;3……}

Ordering and Comparing Integers

Learners need to be able to order Integers.

1. Ensure that the meaning of < and > is clear.
   < - less than (note it looks like an L for Less than)
   > - more than

2. The words 'ascending' (going up, smallest to biggest) and descending' (going down, biggest to smallest) must be made clear to learners.

All four operations on integers need to be done without the use of a calculator. This needs to be practised in class.
General rules for working with signs in Integers:
If the signs are the same: + (add)
If the signs are different: - (subtract)

Teaching Tip: To help learners remember the difference between the < (less than) and > (greater than) sign, point out to them that the < sign looks like an 'L' and 'Less than' also starts with 'L'

Addition and Subtraction of Integers

1. The use of a number line can help learners in this section. A vertical one works better than a horizontal one so that the learners know more easily to move down when subtracting and up when adding. **(Resource 1)**

2. In order to gain a better understanding of a distance from 2 to -6 being 8 a diagram such as the one of the fisherman can also be used. **(Resource 2)**

3. In addition and subtraction, when there are two signs next to each other, these need to be changed into one sign.

Example: 3 + (-2) or 2 + (+1) or 4 – (-3)

+(+)	+
-(-)	+
+(-)	-
-(+)	-

$$3 + (-2) = 3 - 2 = 1$$

$$2 + (+1) = 2 + 1 = 3$$

$$4 - (-3) = 4 + 3 = 7$$

Teaching Tip: As learners often get confused when subtracting from a negative number (such as – 5 – 2), use the word 'owe' in place of the minus sign when explaining it. Owe 5 and owe 2 is usually quite easy to 'see' the answer is 'owe' 7

(- 5 – 2 = - 7)

It can also work with a question like: 6 – 9. Tell the learner he/she has R6 and owes R9. The learners should see that they are short and will therefore still owe

(6 – 9 = - 3)

Multiplication and Division of Integers

1. In Multiplication and Division each number is either a positive or negative, but the same rules apply.

Example: (+10)(+2) or (-5)(+4) or (+3)(-2) or (-10)÷(+5) or (-20) ÷(-4)

Multiplication	Division	Sign
(+)(+)	+/+	+
(-)(-)	-/-	+
(+)(-)	+/-	-
(-)(+)	-/+	-

$$(+10)(+2) = +20$$

$$(-5)(+4) = -20$$

$$(+3)(-2) = -6$$

$$(-10) \div (+5) = -5$$

$$(-20) \div (-4) = 5$$

2. BODMAS is also used in this section when performing a mixture of the four operations on Integers.

Look carefully at this example and ensure that each step is understood

$$\frac{-24}{6} + (3)(-2) - (-2 - 4) + (-3) - (-1)$$

$= -6 \ -6 \ -(-6) - 3 + 1$

$= -6 - 6 + 6 - 3 + 1$

$= -8$

Teaching Tip: Learners often do not understand WHY: positive x negative = negative
or negative x negative = positive

By showing them the following pattern this can help to create a better understanding:

positive x negative:

Count down (and show on the board) in intervals of 3 from 9:

x 3 = 9
3 x 2 = 6
3 x 1 = 3
3 x 0 = 0
3 x –1 = –3
3 x –2 = ?
3 x –3 = ?

Hence the rule: a positive integer x a negative integer = a negative integer

negative x negative:

Using the rule that a positive integer x a negative integer = a negative integer,

established from examples above, the following pattern can be used:

–1 x 3 = –3
–1 x 2 = –2
–1 x 1 = –1
–1 x 0 = 0
–1 x –1 = 1
–1 x –2 = ?
–1 x –3 = ?

Hence the rule: a negative integer x a negative integer = a positive integer

Squares and Cubes of Integers

Remember that squaring a number means multiplying the number by itself.

For example: $5^2 = 5 \times 5 = 25$

As you know, many integers have a negative sign.

When squaring a negative number, the answer will always be positive.

For example: $(-3)^2 = (-3) \times (-3) = 9$

cubing means multiplying a number by the square of the number
(the number will be used 3 times)

For example: $2^3 = 2 \times 2 \times 2 = 8$

$(-3)^3 = (-3) \times (-3) \times (-3) = -27$

Square Roots and Cube Roots of Integers

This is the inverse operation of squaring and cubing. It is like un-doing the operation of squaring or cubing.

Since 5 x 5 = 25, then $\sqrt{25} = 5$

Similarly, 2 x 2 x 2 = 8, then $\sqrt[3]{8} = 2$

The square root of a negative number is not possible and is a non-real number. However, it is possible to find the cube root of a negative number.

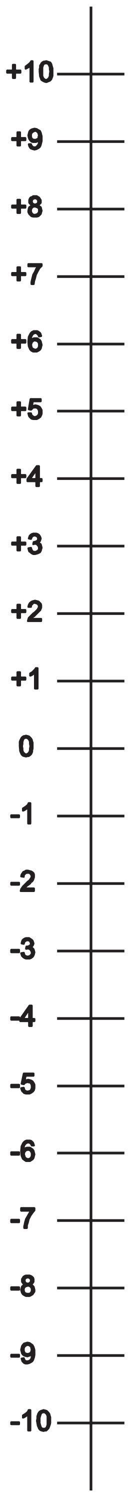
$\sqrt{(-49)}$ = non-real

Since two numbers multiplied together (whether positive or negative) will always be positive.

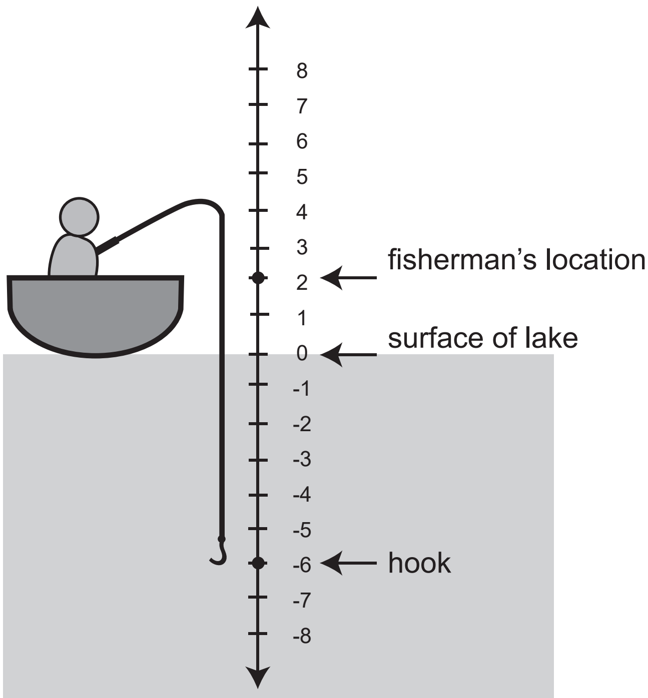
$\sqrt[3]{(-27)} = -3$ (because $(-3)(-3)(-3) = -27$)

RESOURCES

Resource 1 Vertical Number line



Resource 2 Vertical Number line



TOPIC 3: EXPONENTS

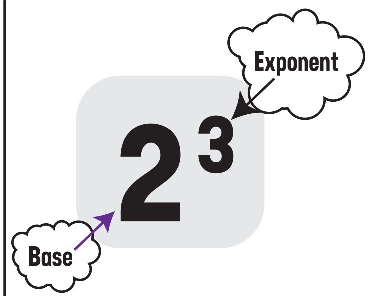
INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area 'Numbers, Operations and Relations' which counts for 25% in the final exam.
- The unit covers the four main laws of Exponents and Scientific Notation.
- It is important to note that this is a topic that learners have difficulty with throughout High School. The time allocation is good so learners should practice as many examples as they can.
- A good understanding of the Laws of Exponents is the key to being good at Algebra which forms the basis of all High School mathematics.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Mental Calculations • Comparing and representing numbers in exponential form • Calculations using numbers in exponential form • Problem solving	• Mental Calculations • Comparing and representing numbers in exponential form (whole numbers, integers and scientific notation) • Calculations using numbers in exponential form (establish and use the general laws of exponents) • Calculate squares, cubes, square roots and cube roots of exponential numbers • Problem solving	• Mental Calculations • Comparing and representing numbers in exponential form (extend scientific notation to include negative exponents) • Calculations using numbers in exponential form (extend the general laws to include integer exponents) • Problem solving

GLOSSARY OF TERMS

Term	Explanation / Diagram
Power	An expression that represents repeated multiplication of the same factor. It is a base raised to an exponent. For example: $3^4 = 3 \times 3 \times 3 \times 3$
Base and Exponent	
Square	To multiply a number by itself Example: $3^2 = 3 \times 3 = 9$
Cube	cubing means multiplying a number by the square of the number (the number will be used 3 times) Example: $4^3 = 4 \times 4 \times 4 = 64$
Square root	The opposite (inverse) operation of squaring Example: $\sqrt{25} = 5$ since 5 x 5 = 25
Cube root	The opposite (inverse) operation of cubing Example: $\sqrt[3]{8} = 2$ since 2 x 2 x 2 = 8
Scientific Notation	A number written as the product of a number between 1 and 10 AND a power of 10 Example: $3{,}42 \times 10^4$ (This represents the number 3 420 000 – 3 million four hundred and twenty thousand)

SUMMARY OF KEY CONCEPTS

Introduction to Exponents

The exponent of a number says how many times the number is multiplied by itself.

In this example: $8^2 = 8 \times 8 = 64$

In words: 8^2 can be read as "8 to the power of 2" or "8 squared".

So using exponents just saves you writing out lots of multiplies!

Laws of Exponents

There are certain rules for exponents that allow us to take 'short cuts'.

Consider the following that will show how each of these rules are derived.

$5^3 \times 5^2$ $= (5 \times 5 \times 5) \times (5 \times 5)$ $= 5 \times 5 \times 5 \times 5 \times 5$ $= 5^5$	$a^2 \times a^3 \times a^4$ $= (a \times a) \times (a \times a \times a) \times (a \times a \times a \times a)$ $= a \times a \times a \times a \times a \times a \times a \times a \times a$ $= a^9$
From these two examples, it can be seen that a quicker way of getting to the solution rather than writing out all the bases then collecting them together would have been to add the exponents.	
$5^3 \times 5^2$ $= 5^{(3+2)}$ $= 5^5$	$a^2 \times a^3 \times a^4$ $= a^{(2+3+4)}$ $= a^9$
Law 1: $a^m \times a^n = a^{m+n}$ When we multiply powers that have the same base, we keep the base and add the exponents	

$4^5 \div 4^2$ $= \frac{(4\times4\times4\times4\times4)}{(4\times4)}$ $= 4\times4\times4$ $= 4^3$	$a^7 \div a^4$ $= \frac{(a\times a\times a\times a\times a\times a\times a)}{(a\times a\times a\times a)}$ $= a^3$
From these two examples, it can be seen that a quicker way of getting to the solution rather than writing out all the bases and simplifying would have been to subtract the exponents.	
$4^5 \div 4^2$ $= 4^{(5-2)}$ $= 4^3$	$a^7 \div a^4$ $= a^{(7-4)}$ $= a^3$
Law 2: $a^m \div a^n = a^{m-n}$ When we divide powers that have the same base, we keep the base and subtract the exponents	

$(3^2)^3$ $= 3^2 \times 3^2 \times 3^2$ $= (3 \times 3) \times (3 \times 3) \times (3 \times 3)$ $= 3^6$	$(b^4)^3$ $= b^4 \times b^4 \times b^4$ $= (b \times b \times b \times b) \times (b \times b \times b \times b) \times (b \times b \times b \times b)$ $= b^{12}$
From these two examples, it can be seen that a quicker way of getting to the solution rather than writing out all the bases and collecting them together would have been to multiply the exponents.	
$(3^2)^3$ $= 3^{(2\times3)}$ $= 3^6$	$(b^4)^3$ $= b^{(4\times3)}$ $= b^{12}$
Law 3: $(a^m)^n = a^{mn}$ When a power is raised to another power, we multiply the exponents	

$(2 \times 3)^3$ $= (2 \times 3) \times (2 \times 3) \times (2 \times 3)$ $= 2 \times 2 \times 2 \times 3 \times 3 \times 3$ $= 2^3\ 3^3$	$(3y)^4$ $= 3y \times 3y \times 3y \times 3y$ $= 3 \times 3 \times 3 \times 3 \times y \times y \times y \times y$ $= 3^4\ y^4$
From these two examples, it can be seen that a quicker way of getting to the solution rather than writing out all the bases and collecting them together would have been to give the exponent to each of the bases.	
$(2 \times 3)^3$ $= 2^3\ 3^3$	$(3y)^4$ $= 3^4\ y^4$
Law 4: $(ab)^m = a^m b^m$ When more than one base is raised to an exponent, the exponent belongs to both (all) bases.	

Note: Law 3 and 4 are often used together.

For example: $(2a^2 b^3)^3 = 2^3 a^6 b^9 = 8a^6 b^9$

Teaching Tip: The Key to the Laws is: Writing all the letters down!!

(If learners are EVER in doubt – tell them to try this)

For example: $(a^2)^3$

Learners may look at a question like this and not be sure of whether to add or multiply the exponents. Remind them that to cube means multiplying a number by the square of the number (the number will be used 3 times). Write this out:

$\therefore$ $(a^2)^3 = a^2 \times a^2 \times a^2$ If they are still unsure at this stage, suggest writing each of the a^2 out in full.

$a^2 \times a^2 \times a^2 = a \times a \times a \times a \times a \times a$

At this stage it should be more clear that $(a^2)^3 = a^6$

Definitions of Exponents

$a^0 = 1$	$5^0 = 1$	Any base (besides zero – that's undefined) To the power of zero is equal to one (To the power of zero really means that there aren't any of those bases left over after simplifying)
$a^{-m} = \frac{1}{a^m}$	2^{-1} $= \frac{1}{2}$	Any base to a negative exponent can be made into a base with a positive exponent by moving the base to the 'other side' of the fraction. (A negative exponent really means that those bases are written on the 'wrong side' of the fraction)

Law 2 helps to show why these work:

Simplify: $\frac{x^5}{x^5}$

Using law 2: $\frac{x^5}{x^5}$ $= x^{(5-5)}$ $= x^0$	Using the longer method of simplifying: $\frac{x^5}{x^5}$ $= 1$ $= \frac{(x.x.x.x.x)}{(x.x.x.x.x)}$ (simplifying of x's)
This demonstration should prove that $x^0 = 1$ Another way to help you remember this rule about a power zero is to say that 'power zero means that there are none of those bases left after simplifying has taken place'. So y^0 means that all y's must have been cancelled out and now there are no y's left at all.	

Law 2 also helps to show WHY the negative exponents can become positive exponents simply by 'moving' them:

Simplify: $\frac{x^2}{x^5}$

Using law 2: $\frac{x^2}{x^5}$ $= x^{(2-5)}$ $= x^{-3}$	Using the longer method of simplifying: $\frac{x^2}{x^5}$ $= \frac{(x.x)}{(x.x.x.x.x)}$ $= \frac{1}{x^3}$ (simplifying of x's)
This demonstration should prove that $x^{-3} = \frac{1}{x^3}$	

Note: These two definitions are not meant to be taught formally in Grade 8 but learners could come across them while simplifying other exponent questions and it would be best to know how to help them gain a better understanding of exponents.

Useful rule/shortcut

If 2 (or more) different bases are raised to the same power

(example $3^2 \times 2^2$ or $5^3 \times 7^3$)

We can multiply the bases together and raise it to that power.

Examples:

$3^2 \times 2^2$	$6^2 = 36$
$5^3 \times 7^3$	35^3

Be careful of the difference between: -3^2 and $(-3)^2$

$-3^2 = (-1)(3x3) = -9$

$(-3)^2 = (-3) \times (-3) = +9$

Useful table for Squares and Square Roots

Perfect square	Square root	Perfect square	Square root
1	$\sqrt{1} = 1$	81	$\sqrt{81} = 9$
4	$\sqrt{4} = 2$	100	$\sqrt{100} = 10$
9	$\sqrt{9} = 3$	121	$\sqrt{121} = 11$
16	$\sqrt{16} = 4$	144	$\sqrt{144} = 12$
25	$\sqrt{25} = 5$	169	$\sqrt{169} = 13$
36	$\sqrt{36} = 6$	196	$\sqrt{196} = 14$
49	$\sqrt{49} = 7$	225	$\sqrt{225} = 15$
64	$\sqrt{64} = 8$		

TOPIC 4: NUMERIC AND GEOMETRIC PATTERNS

INTRODUCTION

- This unit runs for 4.5 hours.
- It is part of the Content Area 'Patterns, Functions and Algebra' which counts for 30% in the final exam.
- The purpose of having an understanding of patterns is to aid in developing mental skills. In order to recognize patterns an understanding of critical thinking and logic are required and these are clearly important skills to develop. Patterns can provide a clear understanding of mathematical relationships. This can be seen in the form of multiplication tables. 2 x 2, 2 x 4, 2 x 6 are just one example of the relationship pattern found in multiplication.
- Understanding patterns helps provide the basis for understanding algebra. This is because a major component of solving algebra problems involves analysing which is related to the understanding of patterns. Without being able to recognize the appearance of patterns the ability to be good at Algebra will be limited.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/ GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Investigate and extend patterns (numeric, geometric) represented visually or in tables • Describe and justify the general rules	• Investigate and extend patterns (numeric, geometric) represented visually or in tables • Algebraic representation of patterns • Describe and justify the general rules in own words and in algebraic language	• Investigate and extend patterns (numeric, geometric) represented visually or in tables • Algebraic representation of patterns • Describe and justify the general rules in own words and in algebraic language

GLOSSARY OF TERMS

Term	Explanation / Diagram
Consecutive	One after the other.
Numeric Sequence	A sequence of numbers that follow a particular pattern.
Constant difference	Same number being added or subtracted to form a sequence.
Geometric Sequence	A sequence of numbers with a constant ratio between consecutive terms.
Constant Ratio	The number used to multiply one term to get to the next term in a geometric sequence. If division occurs, reciprocate and turn it into a multiplication. (Example: $\div 5 = \times \frac{1}{5}$]
Rule	Explanation of how a pattern is formed.
Formula	Instructions in an algebraic form.There will be variables in it.
Term	A number in a given sequence. Example: In the sequence: 5 ; 0 ; -5 ; -10 , all four numbers represent terms and each one of them are in a particular position.
Variable	A letter representing a quantity which is able to have different values.
Position	The place in the sequence held by one of the terms. Example: In the sequence: 2 ; 4 ; 6 ; 8.... 6 is in the 3rd position.

SUMMARY OF KEY CONCEPTS

Describe and extend linear number patterns

1. All of these patterns have a common difference (d) between each term. This is a number that is either added or subtracted each time to find the next term.

2. Learners must be able to describe what is happening in a simple pattern.
   Example: 4 9 14 19…..
   The pattern is formed by adding 5

3. Learners must be able to extend the pattern.
   The above pattern would continue: 24 29 34….

Describe and extend geometric patterns

1. These patterns have common ratio. This is a number that is multiplied each time to find the next term. It could also be a fraction which may seem like division but is essentially the same thing.

2. Learners must be able to describe what is happening in a simple pattern.
   Example: 2 4 8 16
   The pattern is formed by multiplying by 2.

3. Learners must be able to extend the pattern.
   The above pattern would continue: 32 64 128…..

Describe and extend other numeric patterns

1. Learners can be expected to extend and describe patterns that are neither linear nor geometric.

   For example:

   a. 1 ; 4 ; 9 ; 16….

   Learners would be expected to recognise that these are the square numbers and be able to extend the pattern.

   b. 1 ; 1 ; 2 ; 3 ; 5 ; 8…..

   This one is more difficult. From the third term onwards, the terms are found by adding the two previous terms.

   The next two terms would then be: 13 ; 21….

Using tables and rules to extend patterns

A rule is usually written as a formula.

For example: $y = 2x - 5$ or $T_n = -3n + 4$

This rule or formula can then be used to complete a table in which values are given. These values are substituted to find the numbers that form the pattern.

For example: Use the rule $y = 2x + 3$ to complete the table:

x	1	2	3	4	5
y					

$y = 2x + 3$
$= 2(1) + 3$
$= 5$

$y = 2x + 3$
$= 2(2) + 3$
$= 7$

$y = 2x + 3$
$= 2(3) + 3$
$= 9$

$y = 2x + 3$
$= 2(4) + 3$
$= 11$

$y = 2x + 3$
$= 2(5) + 3$
$= 13$

x	1	2	3	4	5
y	5	7	9	11	13

It is important to note the ‘2’ in the rule here and that the common difference is also 2 in the number pattern formed.

Find the rule to describe linear patterns

Linear patterns have a constant difference. This constant difference is essential to finding the rule of a pattern.

For example:
Consider the pattern 4 ; 7 ; 10 ….
The common difference is +3
Consider the pattern 12 ; 7 ; 2….
The common difference is -5

Steps to follow to find the rule of a linear pattern:

- Find the constant difference
- Using the idea that you know what the first term is, multiply the constant
- difference by 1 (representing the first term) and find what still needs to be done to get the first term
- Check the rule works for term 2 and term 3.

For example:

a. Find the rule for the pattern 9 ; 14 ; 19 ; 24…..
Common difference is +5

T_1 = 5(1) + 4 (this will give the 9, the 1st term)
T_2 = 5(2) + 4 (this will give the 14, the 2nd term)
T_3 = 5(3) + 4 (this will give the 19, the 3rd term)
$\therefore$ the rule is to multiply by 5 and add 4.
This can be written as T_n = 5n + 4 where 'n' can represent any position in the pattern.

b. Find the rule and hence the 15th term of the following pattern:
22 ; 19 ; 16 ; 13….
Constant difference: -3

T_1 = -3(1) + 25 (this will give the 22, the 1st term)
T_2 = -3(2) + 25 (this will give the 19, the 2nd term)
T_3 = -3(3) + 25 (this will give the 16, the 3rd term)
$\therefore$ the rule is to multiply by -3 and add 25.
The rule is: $T_n = -3n + 25$

To find the term in the 15th position, substitute 15 for 'n'
T_{15} = -3(15) + 25
= -45 + 25
= -20
The 15th term is -20.

TOPIC 5: FUNCTIONS AND RELATIONSHIPS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area ‘Patterns, Functions and Algebra’ which counts for 30% in the final exam.
- It is important to note that this section is linked closely to Algebra and Equations. Once all three topics have been covered it will be a worthwhile exercise to go back and notice how all skills can be used across all topics.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Input and Output values using flow diagrams, tables and formulae. • Equivalent Forms (verbally, in flow diagrams, in tables, by formulae and by number sentences)	• Input and Output values using flow diagrams, tables, formulae and equations • Equivalent Forms (verbally, in flow diagrams, in tables, by formulae and by equations)	• Input and Output values using flow diagrams, tables, formulae and equations • Equivalent Forms (verbally, in flow diagrams, in tables, by formulae, by equations and by graphs on the Cartesian plane)

GLOSSARY OF TERMS

Term	Explanation / Diagram
Function	A mathematical condition or rule linking the input to the output.
Input	The number/value that was chosen to replace the variable in an expression.
Output	The output is dependent on the input – it is the answer once the operation has been performed according to the expression given.
Equation	A mathematical sentence built from an algebraic expression using an equal sign. Example: $2a + 3 = 10$
Expression	A mathematical formula which can include variables (letters), constants and operations. Example: $2b + 3c$
Flow Diagram	A diagram representing a sequence of movements to be performed on a given value.
Algebraic Rule	An expression representing a rule to be performed on the variable. Example: $3m + 1$ (Multiply the number represented by 'm' by 3 then add 1 to the answer)
Inverse Operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and Division are the inverse operation of each other.

SUMMARY OF KEY CONCEPTS

A function is a special rule or relationship between values

Inputs and Outputs

Every value you put into a function (input) has a specific value that comes out (output) after one or more operations have been performed.
For angles to be adjacent, all three of the above must be evident.

Flow Diagrams

Flow diagrams show how input numbers are changed to become output numbers.

Mathematical rules are used to show what operations have been applied to the input numbers in order to get the output numbers.

A flow diagram is similar to an equation, just written differently.

Example:

Equation	Flow Diagram
5 + 2 = 7	5 → + 2 → = 7

Flow diagrams can be changed into equations when solving is required.

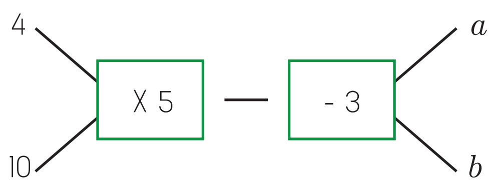
Flow Diagram	Equation
x + 2 = 7	$x + 2 = 7$

Patterns can be represented algebraically. This means that variables (letters that can represent many values) will be used.

Finding output when given the input and the rule

We can use flow diagrams to find missing values.

Example:

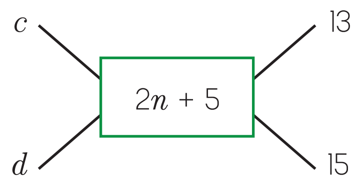


To find 'a': Start with 4, multiply by 5 and then subtract 3 (a = 17)
To find 'b': Start with 10, multiply by 5 and then subtract 3 (b = 47)

Finding input when given the output and the rule

Since we are working backwards, we need to work with INVERSE operations in order to 'undo' the expression.

Example:



First consider the meaning of $2n + 5$: Multiply by 2, then add 5

To find 'c': Using inverse operations:

Start with 13, <u>subtract</u> 5 (inverse operation to addition)

then <u>divide</u> by 2 (inverse operation to multiplication) ($c = 4$)

To find 'd': Using inverse operations:

Start with 15, <u>subtract</u> 5

then <u>divide</u> by 2 ($d = 5$)

Encourage learners to check their answers by working forwards again.

TOPIC 6: ALGEBRAIC EXPRESSIONS

INTRODUCTION

- This unit runs for 4.5 hours.
- It is part of the Content Area ‘Patterns, Functions and Algebra’ which counts for 30% in the final exam.
- The unit covers all the basics of Algebra to lay the groundwork for high school Algebra.
- It is important to note that if learners have a good understanding of the basics of Algebra, their ability to do Algebra in future grades will greatly increase.
- The purpose of this section is to ensure learners understand the basics of Algebra. This will enable them to proceed through all high school Algebra, and have the opportunity to continue with core mathematics until Grade 12. Algebra is not a topic that stands alone, but it is of use to learners in almost all topics covered throughout high school.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Recognise and interpret rules or relationships represented in symbolic form • Identify variables and constants in given formulae or equations	• Add and subtract like terms in expressions • Multiply integers and monomials by: monomials, binomials and trinomials • Divide monomials, binomials and polynomials by integers or monomials • Simplify algebraic expressions • Find squares, square roots, cubes and cube roots of single algebraic terms • Determine the numerical value of an expression using substitution	• Find the product of two binomials • Square a binomial • Factorise algebraic expressions by finding a common factor, difference of two squares and trinomials • Simplify algebraic fractions by using factorisation

GLOSSARY OF TERMS

Term	Explanation / Diagram
Expression	A mathematical statement which can include variables (letters), constants and operations. Example: $2b + 3c$
Product	The answer to a multiplication question.
Sum	The answer to an addition question.
Difference	The answer to a subtraction question.
Quotient	The answer to a division question.
Term	Part of an algebraic expression. Each term is separated by '+' or '-' signs. Example: $3a + 2b$ has two terms.
Coefficient	A number or symbol multiplied with a variable in a term. Example: 3 is the coefficient of y in $3y$ $2a$ is the coefficient of b in the term $2ab$
Variable	Letters of the alphabet which could represent different values. Example: In the expression $m + 2$, m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values.
Constant	A number making up a term on its own in an expression. Example: $a + 3b - 10$: -10 is the constant. Constants cannot change value.
Substitution	Replacing a variable (letter of the alphabet) with a number to perform a calculation. Example: If $b = 3$, the $b + 2 = 3 + 2 = 5$
Monomial	One term expression.
Binomial	Two term expression.
Trinomial	Three term expression.
Polynomial	More than one term expression (two or more).
Like term	Terms that have exactly the same variables. Example: $2a$ and $4a$ are like terms and can be added or subtracted $3abc$ and 10abc are like terms and can be added or subtracted
Unlike term	Terms that do not have exactly the same variables. Example: $3a$ and $2b$ are unlike terms and cannot be added or subtracted x and y are unlike terms and cannot be added or subtracted
Exponent	In the example a^2. The '2' (or squared) is the exponent. It is the number or variable written at the top in smaller font.

SUMMARY OF KEY CONCEPTS

Conventions of Algebra

There are some general conventions (way of doing things) used in algebra that need to be taken note of.

1. Constants are numbers and variables are letters. The constant is always written in front of the variable/s and if there is more than one variable, they need to be written in alphabetical order.
   Examples: a × 2 = 2a
   b × 3 × 4 × a = 12ab

2. If the coefficient of 'a' is 1 or -1, it is not necessary to write it.
   Examples: 1 × x = x
   y × -1 = -y

3. Repeated multiplication can be written in exponential form.
   Examples: 2 × 2 × 2 = 2^3
   $y \times y \times y \times y = y^4$

4. Repeated addition can be written in shorter form using multiplication.
   Examples: 5 + 5 + 5 + 5 = 4(5) (= 20)
   $b + b + b = 3b$

5. Division signs are not usually necessary. Write division as a fraction.
   Examples: $10 \div 5 = \frac{10}{5}$ (= 2)
   $5 \div a = \frac{5}{a}$

6. Multiplication signs are not necessary.
   Examples: 5 × 4 = 5 (4) (= 20)
   $2 \times b = 2b$

7. We can only add and subtract LIKE TERMS.
   Examples: $3a + 4a = 7a$
   $10y - 6y = 4y$
   $5ab + 6a = 5ab + 6a$ (ab and a are NOT Like terms)
   $10a - a = 9a$
   $7x - 6x = x$
   $m - m = 0$

Terms, Coefficients, Constants and Variables

The counting of terms is often seen as an exercise that is not achieving anything. In Grade 9, learners begin to factorise which can become quite difficult. If they are good at counting terms and knowing immediately how many terms an expression has, they will have a good grounding to learn how to factorise.

1. Algebraic expressions are made up of terms.
   - Terms are separated by +/- signs.
   - However, any expression inside a bracket is one term only AND any expression over a divide line is one term only.

2. A coefficient is a number in front of the variable – INCLUDING the sign.
   - Remember: "The sign to the left of the term belongs to it". If there is no sign, it is positive.

3. A variable is a letter of the alphabet that can represent an unknown value.

4. A constant is a term that is made up of a number only with no variables.

If an expression has:	It is called:
One term	Monomial
Two terms	Binomial
Three terms	Trinomial
Multiple terms (two or more)	Polynomial

Examples: 4x2 - 3x + 2

- The expression has three terms (it can be called a trinomial or a polynomial).
- The constant is 2
- The coefficient of x^2 is 4
- The coefficient of x is -3

$5(a+b) - \frac{2a}{3} + \frac{b}{2} - 5$

- The expression has four terms (a polynomial).
- The constant is -5
- The coefficient of a is $\frac{-2}{3}$
- The coefficient of b is $\frac{1}{2}$
- The coefficient of (a + b) is 5

Teaching Tip: In order to assist learners in counting terms in a long expression, show them by following these steps:

- Using a pencil, circle all plus and minus signs (in the example below there are now 4 circled)
- Erase anything that has been circled that is either inside a bracket or over a divide line (in the example below, two have been erased)
- Draw a line through all signs that are still circled
- These lines are dividing the terms up (the two lines in the example below show that there are <u>three</u> terms in this expression)

$$\frac{2a + 3}{4} - (2a + b) + 5ab$$

$$\frac{2a + 3}{4} - (2a + b) + 5ab$$

$$\frac{2a + 3}{4} - (2a + b) + 5ab$$

Simplification of Expressions

The counting of terms is often seen as an exercise that is not achieving

anything. In Grade 9, learners begin to factorise which can become quite difficult. If they are good at counting terms and knowing immediately how many terms an expression has, they will have a good grounding to learn how to factorise.

Remember: We can ONLY add and subtract LIKE terms;
The 'names' of terms don't change, only the coefficient does
(AND the sign to the left of a term belongs to that term!)

For example:

a. $5x + 2x = 7x$ (new coefficient but x remains the same)

b. $6y - 2y = 4y$

c. $7x + 4y = 7x + 4y$ (unlike terms so can't add or subtract)

d. $5ab - 4a + 2a + 3ab = 8ab - 2a$

Teaching Tip: Where possible try and use real life examples when practicing the addition and subtraction of like terms. For example, ask learners what 3 books + 4 books is. They will probably answer correctly, i.e.: 7 books. ($3b + 4b = 7b^2$ is a common error when learners first start Algebra, but if you say 3 books + 7 books, they are unlikely to make this mistake). Using real life examples should help them to see that a 'name' doesn't change when adding and subtracting like terms.

Teaching Tip: Using the same idea you can also show that unlike terms cannot be added or subtracted. For example, ask learners what 3 pencils + 4 cars is. They should see that this would have to remain 3 pencils + 4 cars and will therefore have a better understanding that they cannot add and subtract terms unless they are exactly the same.

TOPIC 7: ALGEBRAIC EQUATIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area 'Patterns, Functions and Algebra' which counts for 30% in the final exam.
- The unit covers the solving of basic Equations.
- It is important to note that learners must master the skills of solving simple Equations. If not, they will not be able to move up a level later in the year. Grade 9 to 11 have more difficult Equations and the learners will not manage these.
- The purpose of learning to solve Equations is to help with Problem Solving which is the basis of all mathematics.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Write number sentences to describe problem situations • Solve and complete number sentences by inspection and trial & improvement • Determine the numerical value of an expression by substitution • Identify variables and constants	• Use substitution in equations to generate tables of ordered pairs • Use additive and multiplicative inverses to solve equations • Use the laws of exponents to solve equations	• Use factorisation to solve equations • Solve equations of the form: a product of factors = 0

GLOSSARY OF TERMS

Term	Explanation / Diagram
Equation	A mathematical statement with an equal sign that includes a variable. Example: $3x - 5 = 20$
Expression	An algebraic statement consisting of terms with variables and constants. There is no equal sign. Example: $3a + 2b$
Formula	A formula is used to calculate a specific type of answer and has variables that represent a certain kind of value. Example: Area = l × b This formula finds area of a rectangle and only measurements can replace the l and b.
Variable	Letters of the alphabet which could represent different values. Example: In the expression m + 2, m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values.
Like Terms	Terms that have exactly the same variables. Example: $2a$ and $4a$ are like terms and can be added or subtracted $3abc$ and $10abc$ are like terms and can be added or subtracted
Inverse Operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and Division are the inverse operation of each other.

SUMMARY OF KEY CONCEPTS

What is an Equation?

An equation is a mathematical statement that two things are equal. The most common equations contain one or more variables. These are the type that will be dealt with in Grade 8.

Example of an equation: $2a + 6 = 10$

1. The Left Hand Side (LHS) = Right Hand Side (RHS)

2. An equation can be solved – the value of the variable that makes the equation true is found.

   Sometimes, the equations are fairly simple and it is possible to know the answer without doing a big calculation – this is called 'solving by inspection'.

Equations will not always be fairly simple ones so it is essential to learn how to solve them using algebraic methods.

Solving Equations Algebraically

In order to do this, a learner needs to:

1. Understand BODMAS and how to 'undo' operations (as was done in the Functions and Relationships section while looking for input value when you had the output value)

2. Be able to collect like terms (as was done in the Algebra section)

The most important rule to keep in mind when dealing with equations:

Whatever you do to one side of an equation you must ALWAYS do to the other side in order to keep the equation balanced (Note teaching tip and resource below)

For example:

1. $a + 2 = 10$ the aim of solving the equation: find what 'a' needs to be to make this equation true ' + 2' is in the way of getting 'a' alone; the inverse operation of + 2 is – 2, therefore we need to subtract 2 from BOTH sides to ensure it stays balanced

   $a + 2 \underline{-2} = 10 \underline{-2}$

   $a = 8$ (like terms were collected on each side)

2. $3x - 4 = 11$ — the aim: to get x on its own and find its value
Consider the order of operations to be performed on x then ‘undo’ them by using inverse operations

First we need to ‘undo’ – 4 (so we will + 4)

$3x - 4 + 4 = 11 + 4$

$\frac{3x}{3} = \frac{15}{3}$ — Then we need to ‘undo’ x 3 (so we will ÷ 3)

$3x = 15$

$x = 5$

An answer can be checked by putting it back into the equation and checking if the

LHS = RHS

Check:

LHS = $3x - 4$ RHS = 11

= 3 (5) – 4

= 15 – 4

= 11 = RHS LHS = RHS $\therefore x = 5$ is correct

Teaching Tip: When teaching how to solve Equations algebraically (using inverse operations) tell the learners (or even show if possible) to picture a scale **(Resource 1)**. An equation has an equal sign which makes it perfectly balanced. If something is added to one side only it will no longer be perfectly balanced **(Resource 2)**. Therefore, to keep it balanced, whatever is added to one side should always be added to the other as well. Similarly, if multiplying by a number on one side is needed, then the other side should also be multiplied etc.

Encourage learners to use another colour if possible when showing what they are doing to both sides. So in Example 2 above, the +4 on each side would be in a colour other than the one they are working with and so would ÷3 on each side.

Encourage learners to discuss with you and fellow learners the operation being used on the variable and what the inverse operation will be in order to solve for the variable.

Note: Refer back to the ‘Functions and Relationships’ topic once learners have solved some equations and point out the connection between solving equations and finding the input given the output.

RESOURCES

Resource 1 (Page 43) Balanced Scale

Resource 2 (Page 43) Unbalanced Scale